

QUARTIC POWER SERIES IN $\mathbb{F}_3((T^{-1}))$ WITH BOUNDED PARTIAL QUOTIENTS, II

by

Domingo Gomez (Universidad de Cantabria)
and Alain Lasjaunias (Université Bordeaux 1)

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1. Introduction and result

We are concerned with power series in $1/T$ over a finite field, where T is an indeterminate. If the base field is \mathbb{F}_q , the finite field with q elements, these power series belong to the field $\mathbb{F}_q((T^{-1}))$, which will be here denoted by $\mathbb{F}(q)$. Thus a nonzero element of $\mathbb{F}(q)$ is represented by

$$\alpha = \sum_{k \leq k_0} u_k T^k \quad \text{where } k_0 \in \mathbb{Z}, u_k \in \mathbb{F}_q \quad \text{and } u_{k_0} \neq 0.$$

An absolute value on this field is defined by $|\alpha| = |T|^{k_0}$ where $|T| > 1$ is a fixed real number. We also denote by $\mathbb{F}(q)^+$ the subset of power series α such that $|\alpha| > 1$. We know that each irrational element $\alpha \in \mathbb{F}(q)^+$ can be expanded as an infinite continued fraction. This is denoted

$$\alpha = [a_1, a_2, \dots, a_n, \dots] \quad \text{where } a_i \in \mathbb{F}_q[T] \text{ and } \deg(a_i) > 0 \text{ for } i \geq 1.$$

By truncating this expansion we obtain a rational element, called a convergent to α and denoted by x_n/y_n for $n \geq 1$. The polynomials $(x_n)_{n \geq 0}$ and $(y_n)_{n \geq 0}$, called continuants, are both defined by the same recursion formula : $K_n = a_n K_{n-1} + K_{n-2}$ for $n \geq 2$, with the initials conditions $x_0 = 1$ and $x_1 = a_1$ or $y_0 = 0$ and $y_1 = 1$. The polynomials a_i are called the partial quotients of the expansion. For $n \geq 1$, we denote $\alpha_{n+1} = [a_{n+1}, a_{n+2}, \dots]$, called the complete quotient, and we have

$$\alpha = [a_1, a_2, \dots, a_n, \alpha_{n+1}] = (x_n \alpha_{n+1} + x_{n-1}) / (y_n \alpha_{n+1} + y_{n-1}).$$

The reader may consult [S] for a general account on continued fractions in power series fields. Throughout this note we are considering the case $q = 3$.

In a previous article [L1], the second author of this note investigated the existence of particular power series in $\mathbb{F}(3)$, algebraic over $\mathbb{F}_3[T]$, having all partial quotients of degree 1 in their continued fraction expansion. A first example of such algebraic power series appeared in [MR] (p. 401-402). In this article [MR], Mills and Robbins, developing the pioneer work by Baum and Sweet [BS], introduced in the general case a particular subset of algebraic power series. These power series, now called hyperquadratic, are irrational elements α satisfying an equation $\alpha = f(\alpha^r)$ where r is a power of the characteristic of the base field and f is a linear fractional transformation with integer (polynomials in T) coefficients.

The theorem which we present in this note is an extended version of the one presented in [L1]. The proof given here is based on a method developed in [L2]. Actually, this method could be applied to obtain other hyperquadratic continued fractions with all partial quotients of degree 1 in other fields $\mathbb{F}(p)$, for any odd prime p . We have the following :

Theorem. *Let $m \in \mathbb{N}^*$, $\boldsymbol{\eta} = (\eta_1, \eta_2, \dots, \eta_m) \in (\mathbb{F}_3^*)^m$ where $\eta_m = (-1)^{m-1}$ and $\mathbf{k} = (k_1, k_2, \dots, k_m) \in \mathbb{N}^m$ where $k_1 \geq 2$ and $k_{i+1} - k_i \geq 2$ for $i = 1, \dots, m-1$. We define the following integers,*

$$t_{i,n} = k_m(3^n - 1)/2 + k_i 3^n \quad \text{for } 1 \leq i \leq m \quad \text{and } n \geq 0.$$

We define two sequences $(\lambda_t)_{t \geq 1}$ and $(\mu_t)_{t \geq 1}$ in \mathbb{F}_3 . For $n \geq 0$, we have

$$\lambda_t = \begin{cases} 1 & \text{if } 1 \leq t \leq t_{1,0}, \\ (-1)^{mn+i} & \text{if } t_{i,n} < t \leq t_{i+1,n} \quad \text{for } 1 \leq i < m, \\ (-1)^{m(n+1)} & \text{if } t_{m,n} < t \leq t_{1,n+1}. \end{cases}$$

Also $\mu_1 = 1$ and for $n \geq 0$, $1 \leq i \leq m$ and $t > 1$

$$\mu_t = \begin{cases} (-1)^{n(m+1)} \eta_i & \text{if } t = t_{i,n} \text{ or } t = t_{i,n} + 1, \\ 0 & \text{otherwise.} \end{cases}$$

Let $\omega(m, \boldsymbol{\eta}, \mathbf{k}) \in \mathbb{F}(3)$ be defined by the infinite continued fraction expansion

$$\omega(m, \boldsymbol{\eta}, \mathbf{k}) = [a_1, a_2, \dots, a_n, \dots] \quad \text{where} \quad a_n = \lambda_n T + \mu_n.$$

We set $l = 1 + k_m$ and we consider the two usual sequences $(x_n)_{n \geq 0}$ and $(y_n)_{n \geq 0}$ as being the numerators and denominators of the convergents to $\omega(m, \boldsymbol{\eta}, \mathbf{k})$.

Then $\omega(m, \boldsymbol{\eta}, \mathbf{k})$ is the unique root in $\mathbb{F}(3)^+$ of the quartic equation

$$X = \frac{x_l X^3 + (-1)^{m-1} x_{l-3}}{y_l X^3 + (-1)^{m-1} y_{l-3}}.$$

Remark. The case $m = 1$ and thus $\boldsymbol{\eta} = (1)$, $\mathbf{k} = (k_1)$, of this theorem is proved in [L1]. The case $m = 2$, $\boldsymbol{\eta} = (-1, -1)$ and $\mathbf{k} = (3, 6)$ corresponds to the example which was introduced by W. Mills and D. Robbins, in [MR].

2. Proof of the theorem

The proof will be divided into three steps.

• *First step of the proof:* According to [L2, Theorem 1, p. 332], there exists a unique element $\beta \in \mathbb{F}(3)$ defined by $\beta = [a_1, \dots, a_l, \beta_{l+1}]$ and satisfying

$$\beta^3 = (-1)^m (T^2 + 1) \beta_{l+1} + T + 1 \quad \text{and} \quad a_i = \lambda_i T + \mu_i, \text{ for } 1 \leq i \leq l,$$

where λ_i, μ_i are the elements defined in the theorem. We know that this element is hyperquadratic and that it is the unique root in $\mathbb{F}(3)^+$ of the algebraic equation $X = (x_l X^3 + B)/(y_l X^3 + D)$ where

$$B = (-1)^m (T^2 + 1) x_{l-1} - (T + 1) x_l \quad \text{and} \quad D = (-1)^m (T^2 + 1) y_{l-1} - (T + 1) y_l.$$

We need to transform B and D . Using the recursive formulas for the continuants, we can write

$$K_{l-3} = (a_l a_{l-1} + 1) K_{l-1} - a_{l-1} K_l. \tag{1}$$

The l first partial quotients of β are given, from the hypothesis of the theorem, and we have

$$a_{l-1} = (-1)^{m-1}(T+1) \quad \text{and} \quad a_l = (-1)^{m-1}(-T+1). \quad (2)$$

Combining (1), applied to both sequences x and y , and (2), we get

$$B = (-1)^{m-1}x_{l-3} \quad \text{and} \quad D = (-1)^{m-1}y_{l-3}.$$

Hence we see that β is the unique root in $\mathbb{F}(3)^+$ of the quartic equation stated in the theorem.

• *Second step of the proof:* In this section $l \geq 1$ is a given integer. We consider all the infinite continued fractions $\alpha \in \mathbb{F}(3)$ defined by $\alpha = [a_1, \dots, a_l, \alpha_{l+1}]$ where

$$(I) \quad : \quad a_i = \lambda_i T + \mu_i \quad \text{with} \quad (\lambda_i, \mu_i) \in \mathbb{F}_3^* \times \mathbb{F}_3, \quad \text{for} \quad 1 \leq i \leq l$$

and

$$(II) \quad : \quad \alpha^3 = \epsilon_1(T^2 + 1)\alpha_{l+1} + \epsilon_2 T + \nu_0 \quad \text{with} \quad (\epsilon_1, \epsilon_2, \nu_0) \in \mathbb{F}_3^* \times \mathbb{F}_3^* \times \mathbb{F}_3.$$

See [L2, Theorem 1, p. 332], for the existence and unicity of $\alpha \in \mathbb{F}(3)$ defined by the above relations. Our aim is to show that these continued fraction expansions can be explicitly described, under particular conditions on the parameters $(\lambda_i, \mu_i)_{1 \leq i \leq l}$ and $(\epsilon_1, \epsilon_2, \nu_0)$. Following the same method as in [L2], we first prove :

Lemma 1. *Let $(\lambda, \epsilon_1, \epsilon_2) \in (\mathbb{F}_3^*)^3$ and $\nu \in \mathbb{F}_3$. We set $U = \lambda T^3 - \epsilon_2 T + \nu$, and $V = \epsilon_1(T^2 + 1)$. We set $\delta = \lambda + \epsilon_2$ and we assume that $\delta \neq 0$. We define $\epsilon^* = 1$ if $\nu = 0$ and $\epsilon^* = -1$ if $\nu \neq 0$. Then the continued fraction expansion for U/V is given by*

$$U/V = [\epsilon_1 \lambda T, -\epsilon_1(\delta T + \nu), -\epsilon_1(\epsilon^* \delta T + \nu)].$$

Moreover, setting $U/V = [u_1, u_2, u_3]$, then for $X \in \mathbb{F}(3)$ we have

$$[U/V, X] = [u_1, u_2, u_3, \frac{X}{(T^2 + 1)^2} + \frac{\epsilon^* \epsilon_1(\delta T + \nu)}{T^2 + 1}].$$

Proof : Since $\epsilon_1^2 = 1$ and $\delta^2 = 1$, we can write

$$U = \epsilon_1 \lambda T V - \delta T + \nu \quad \text{and} \quad V = \epsilon_1 (\delta T + \nu) (\delta T - \nu) + \epsilon_1 (1 + \nu^2). \quad (3)$$

Clearly (3) implies the following continued fraction expansion

$$U/V = [\epsilon_1 \lambda T, -\epsilon_1 (\delta T + \nu), \epsilon_1 (1 + \nu^2) (-\delta T + \nu)]. \quad (4)$$

Finally, observing that $\epsilon_1 (1 + \nu^2) = \epsilon^* \epsilon_1$ and $\epsilon^* \epsilon_1 \nu = -\epsilon_1 \nu$, we see that (4) is the expansion stated in the lemma. The last formula is obtained from [L2] (Lemma 3.1 p. 336). According to this lemma, we have

$$[U/V, X] = [u_1, u_2, u_3, X'] \quad \text{where} \quad X' = X(u_2 u_3 + 1)^{-2} - u_2 (u_2 u_3 + 1)^{-1}.$$

We check that $u_2 u_3 = T^2$ if $\nu = 0$ and $u_2 u_3 = \nu^2 - T^2$ if $\nu \neq 0$, therefore we have $u_2 u_3 + 1 = \epsilon^* (T^2 + 1)$ and this implies the desired equality.

We shall prove now a second lemma. In the sequel we define $f(n)$ as $3n + l - 2$ for $n \geq 1$. We have the following :

Lemma 2. *Let $\alpha = [a_1, \dots, a_n, \dots]$ be an irrational element of $\mathbb{F}(3)$. We assume that for an index $n \geq 1$ we have $a_n = \lambda_n T + \mu_n$ with $(\lambda_n, \mu_n) \in \mathbb{F}_3^* \times \mathbb{F}_3$ and*

$$\alpha_n^3 = \epsilon_1 (T^2 + 1) \alpha_{f(n)} + \epsilon_{2,n} T + \nu_{n-1} \quad \text{where} \quad (\epsilon_1, \epsilon_{2,n}, \nu_{n-1}) \in (\mathbb{F}_3^*)^2 \times \mathbb{F}_3.$$

We set $\nu_n = \mu_n - \nu_{n-1}$ and $\epsilon_n^ = 1$ if $\nu_n = 0$ or $\epsilon_n^* = -1$ if $\nu_n \neq 0$. We set $\delta_n = \lambda_n + \epsilon_{2,n}$, and $\epsilon_{2,n+1} = -\epsilon_n^* \delta_n$. We assume that $\delta_n \neq 0$. Then we have :*

$$(a_{f(n)}, a_{f(n)+1}, a_{f(n)+2}) = (\epsilon_1 \lambda_n T, -\epsilon_1 (\delta_n T + \nu_n), -\epsilon_1 (\epsilon_n^* \delta_n T + \nu_n))$$

and

$$\alpha_{n+1}^3 = \epsilon_1 (T^2 + 1) \alpha_{f(n+1)} + \epsilon_{2,n+1} T + \nu_n.$$

Proof: We can write $\alpha_n^3 = [a_n^3, \alpha_{n+1}^3] = [\lambda_n T^3 + \mu_n, \alpha_{n+1}^3]$. Consequently

$$\alpha_n^3 = \epsilon_1 (T^2 + 1) \alpha_{f(n)} + \epsilon_{2,n} T + \nu_{n-1}$$

is equivalent to

$$[(\lambda_n T^3 + \mu_n - \epsilon_{2,n} T - \nu_{n-1}) / (\epsilon_1 (T^2 + 1)), \epsilon_1 (T^2 + 1) \alpha_{n+1}^3] = \alpha_{f(n)}. \quad (5)$$

Now we apply Lemma 1 with $U = \lambda_n T^3 - \epsilon_{2,n} T + \nu_n$ and $X = \epsilon_1 (T^2 + 1) \alpha_{n+1}^3$. Consequently (5) can be written as

$$[\epsilon_1 \lambda_n T, -\epsilon_1 (\delta_n T + \nu_n), -\epsilon_1 (\epsilon_n^* \delta_n T + \nu_n), X'] = \alpha_{f(n)} \quad (6)$$

where

$$X' = (\epsilon_1 \alpha_{n+1}^3 + \epsilon_1 \epsilon_n^* (\delta_n T + \nu_n)) / (T^2 + 1). \quad (7)$$

Moreover we have $|\alpha_{n+1}^3| \geq |T^3|$ and consequently $|X'| > 1$. Thus (6) implies that the three partial quotients $a_{f(n)}$, $a_{f(n)+1}$ and $a_{f(n)+2}$ are as stated in this lemma and also that we have $X' = \alpha_{f(n+1)}$. Combining this last equality with (7), and observing that $-\epsilon_n^* \nu_n = \nu_n$, we obtain the desired statement.

Applying Lemma 2, we see that for a continued fraction defined by (I) and (II), the partial quotients, from the rank $l+1$ onward, can be given explicitly three by three, as long as the quantity δ_n is not zero. This is taken up in the following proposition :

Proposition 3. *Let $\alpha \in \mathbb{F}(3)$ be an infinite continued fraction expansion defined by (I) and (II). Then there exists $N \in \mathbb{N}^* \cup \{\infty\}$ such that*

$$a_n = \lambda_n T + \mu_n \quad \text{where} \quad (\lambda_n, \mu_n) \in \mathbb{F}_3^* \times \mathbb{F}_3 \quad \text{for} \quad 1 \leq n < f(N).$$

For $1 \leq n < f(N)$, we define $\nu_n = \sum_{1 \leq i \leq n} (-1)^{n-i} \mu_i + (-1)^n \nu_0$. Then we have

$$\mu_{f(n)} = 0 \quad \text{and} \quad \mu_{f(n)+1} = \mu_{f(n)+2} = -\epsilon_1 \nu_n \quad \text{for} \quad 1 \leq n < N.$$

For $1 \leq n < N$, we define $\epsilon_n^* = 1$ if $\nu_n = 0$ or $\epsilon_n^* = -1$ if $\nu_n \neq 0$. We define recursively the sequence $(\delta_n)_{1 \leq n \leq N}$ by

$$\delta_1 = \lambda_1 + \epsilon_2 \quad \text{and} \quad \delta_n = \lambda_n - \epsilon_{n-1}^* \delta_{n-1} \quad \text{for} \quad 2 \leq n \leq N.$$

Then, for $1 \leq n < N$, we have

$$\lambda_{f(n)} = \epsilon_1 \lambda_n, \quad \lambda_{f(n)+1} = -\epsilon_1 \delta_n \quad \text{and} \quad \lambda_{f(n)+2} = -\epsilon_1 \epsilon_n^* \delta_n.$$

Proof: Starting from (II), since $f(1) = l+1$, setting $\epsilon_2 = \epsilon_{2,1}$ and observing that all the partial quotients are of degree 1, we can apply repeatedly Lemma

2 as long as we have $\delta_n \neq 0$. If δ_n happens to vanish, the process is stopped and we denote by N the first index such that $\delta_N = 0$, otherwise N is ∞ . From the formulas $\delta_n = \lambda_n + \epsilon_{2,n}$ and $\epsilon_{2,n+1} = -\epsilon_n^* \delta_n$ for $n \geq 1$, we obtain the recursive formulas for the sequence δ . Moreover, the formula $\nu_n = \mu_n - \nu_{n-1}$, implies clearly the equality for ν_n . Finally the formulas concerning μ and λ are directly derived from the three partial quotients $a_{f(n)}$, $a_{f(n)+1}$ and $a_{f(n)+2}$ given in Lemma 2.

• *Last step of the proof:* We start from the element $\beta \in \mathbb{F}(3)$, introduced in the first step of the proof, defined by its l first partial quotients, where $l = k_m + 1$, and by (II) with $(\epsilon_1, \epsilon_2, \nu_0) = ((-1)^m, 1, 1)$. According to the first step of the proof, we need to show that $\beta = \omega(m, \boldsymbol{\eta}, \mathbf{k})$. To do so, we apply Proposition 3 to β , and we show that $N = \infty$ and that the resulting sequences $(\lambda_n)_{n \geq 1}$ and $(\mu_n)_{n \geq 1}$ are the one which are described in the theorem. From the l -tuple (μ_1, \dots, μ_l) and $\nu_0 = 1$, we obtain

$$\nu_t = \eta_i \quad \text{if} \quad t = t_{i,0} \quad \text{and} \quad \nu_t = 0 \quad \text{otherwise,} \quad \text{for} \quad 1 \leq t \leq l. \quad (8)$$

Since $\mu_{f(n)+1} = \mu_{f(n)+2}$, we have $\nu_{f(n)+2} = \nu_{f(n)}$. Since $\mu_{f(n)} = 0$, we also have $\nu_{f(n)} = -\nu_{f(n)-1} = -\nu_{f(n-1)+2}$. This implies $\nu_{f(n)+2} = (-1)^{n-1} \nu_{f(1)+2}$. Since $\nu_{f(1)+2} = \nu_{f(1)} = -\nu_{f(1)-1} = -\nu_l = 0$, we obtain

$$\nu_{f(n)} = \nu_{f(n)+2} = 0 \quad \text{for} \quad 1 \leq n < N. \quad (9)$$

Moreover, from $\nu_{f(n)+1} = \mu_{f(n)+1} - \nu_{f(n)}$ and (9), we also get

$$\nu_{f(n)+1} = -\epsilon_1 \nu_n \quad \text{for} \quad 1 \leq n < N. \quad (10)$$

Now, it is easy to check that we have $f(t_{i,n}) + 1 = t_{i,n+1}$. Since $\epsilon_1 = (-1)^m$, (10) implies $\nu_{t_{i,n}} = (-1)^{m+1} \nu_{t_{i,n-1}}$ if $t_{i,n} < f(N)$. By induction from (8), with (9) and (10), we obtain

$$\nu_{t_{i,n}} = (-1)^{(m+1)n} \eta_i \quad \text{and} \quad \nu_t = 0 \quad \text{if} \quad t \neq t_{i,n}, \quad \text{for} \quad 1 \leq t < f(N). \quad (11)$$

Since we have $\mu_n = \nu_n + \nu_{n-1}$, from (11) and $\nu_0 = 1$, we see that μ_n satisfies the formulas given in the theorem, for $1 \leq n < f(N)$. Moreover, (11) implies

clearly the following :

$$\epsilon_t^* = \begin{cases} -1 & \text{if } t = t_{i,n}, \\ 1 & \text{otherwise} \end{cases} \quad \text{for } 1 \leq t < f(N). \quad (12)$$

Now we turn to the definition of the sequence $(\lambda_n)_{n \geq 1}$ given in the theorem, corresponding to the element ω . With our notations and according to (12), we observe that this definition can be translated into the following formulas

$$\lambda_1 = 1 \quad \text{and} \quad \lambda_n = \epsilon_{n-1}^* \lambda_{n-1} \quad \text{for } 2 \leq n < f(N). \quad (13)$$

Consequently, to complete the proof, we need to establish that $N = \infty$ and that (13) holds. The recurrence relation binding the sequences δ and λ , introduced in Proposition 3, can be written as

$$\delta_n + \lambda_n = -\epsilon_{n-1}^* (\delta_{n-1} + \lambda_{n-1}) + \epsilon_{n-1}^* \lambda_{n-1} - \lambda_n \quad \text{for } 2 \leq n \leq N. \quad (14)$$

Comparing (13) and (14), we see that $\delta_n + \lambda_n = 0$, for $n \geq 1$, will imply that δ_n never vanishes, i.e. $N = \infty$, and that the sequence $(\lambda_n)_{n \geq 1}$ is the one which is described in the theorem. So we only need to prove that $\delta = -\lambda$. Since β and ω have the same first partial quotients, (13) holds for $2 \leq n \leq l$. Since $\delta_1 = \lambda_1 + \epsilon_2 = -1 = -\lambda_1$, combining (13) and (14), we obtain $\delta_n = -\lambda_n$ for $1 \leq n \leq l$. We also have $\lambda_{l+1} = \lambda_{f(1)} = \epsilon_1 \lambda_1 = (-1)^m = \lambda_l$, and therefore we get $\delta_{l+1} = \lambda_{l+1} - \epsilon_l^* \delta_l = \lambda_{l+1} + \lambda_l = -\lambda_{l+1}$. By induction, we shall now prove that $\delta_t = -\lambda_t$ for $t = f(n) + 1, f(n) + 2$ and $f(n + 1)$ with $n \geq 1$. From (9) and (10), we have $\epsilon_{f(n)}^* = \epsilon_{f(n)+2}^* = 1$ and $\epsilon_{f(n)+1}^* = \epsilon_n^*$. Thus we get

$$\delta_{f(n)+1} = \lambda_{f(n)+1} - \epsilon_{f(n)}^* \delta_{f(n)} = \lambda_{f(n)+1} + \lambda_{f(n)} = -\epsilon_1 \delta_n + \epsilon_1 \lambda_n = -\lambda_{f(n)+1}.$$

$$\delta_{f(n)+2} = \lambda_{f(n)+2} - \epsilon_{f(n)+1}^* \delta_{f(n)+1} = \lambda_{f(n)+2} + \epsilon_n^* \lambda_{f(n)+1} = -\lambda_{f(n)+2}.$$

$$\begin{aligned} \delta_{f(n+1)} &= \lambda_{f(n+1)} - \epsilon_{f(n)+2}^* \delta_{f(n)+2} = \epsilon_1 \lambda_{n+1} + \lambda_{f(n)+2} = \epsilon_1 (\lambda_{n+1} - \epsilon_n^* \delta_n) \\ &= \epsilon_1 \delta_{n+1} = -\epsilon_1 \lambda_{n+1} = -\lambda_{f(n+1)}. \end{aligned}$$

So the proof of the theorem is complete.

Finally, we make a remark concerning the apparent peculiarity of the theorem which is proved in this note. We make the following conjecture: Let $\alpha \in \mathbb{F}(3)$ be a hyperquadratic element, which is not quadratic; then α has all its partial quotients of degree 1, except for the first ones, if and only if there exist a linear fractional transformation f , with coefficients in $\mathbb{F}_3[T]$ and determinant in \mathbb{F}_3^* , a triple $(m, \boldsymbol{\eta}, \mathbf{k})$ and a pair $(\lambda, \mu) \in \mathbb{F}_3^* \times \mathbb{F}_3$ such that $\alpha(T) = f(\omega(m, \boldsymbol{\eta}, \mathbf{k})(\lambda T + \mu))$.

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Domingo Gomez Universidad de Cantabria Departamento de Matemáticas 39005 Santander, Spain Domingo.Gomez@unican.es	Alain Lasjaunias Université Bordeaux 1 C.N.R.S.-UMR 5251 33405 Talence, France Alain.Lasjaunias@math.u-bordeaux1.fr
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